

The relationship between the fracture properties, tensile strength and critical stress intensity factor of organic solids and their molecular structure

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Abstract

For a variety of pharmaceutical solids it has been shown that both tensile strength, σ_T , and critical stress intensity factor, K_{IC} , determined by beam bending are related to cohesive energy density by a simple factor. The equations allow the prediction of both mechanical properties from a knowledge of the chemical structure of the material.

Keywords: Tensile strength; Critical stress intensity factor; Cohesive energy density; Organic crystal

1. Introduction

Recently several papers have shown that there is a relationship between the mechanical properties of organic crystals and their molecular properties, as represented by their cohesive energy density (CED) as derived from solubility parameters (δ). Of specific interest are the papers by Roberts et al. (1991) who showed that cohesive energy density calculated from chemical structure could be used to estimate Young's modulus of elasticity of a variety of pharmaceutical powders and Roberts et al. (1993a), who demonstrated a relationship between the indentation hardness of some organic crystals and their cohesive energy density together with parameters from their crys-

tal structure. If slip planes could be identified from the crystal structure it was possible to estimate indentation hardness with a reasonable accuracy.

More recently, Newton et al. (1993) have extended the approach to the fracture properties of a homologous series of benzoic acid esters indicated by tensile strength (σ_T), critical stress intensity factors (K_{IC}) and strain energy release rate (G_{IC}), showing, in general, that both tensile strength and critical stress intensity factor increase with an increase in cohesive energy density.

In this paper the relationship between the cohesive energy density and the tensile strength and critical stress intensity factors (both measured by beam bending) of a variety of pharmaceutical solids (both excipients and drug mole-

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Table 1
True densities, molar volumes, solubility parameters and cohesive energy densities of the various organic solids

Material	True density (g cm ⁻³)	V (cm ³ mol ⁻¹)	δ (MPa ^{1/2})	CED (MPa)
Testosterone propionate	1.13	294.0	19.4	376.4 ^a
Ibuprofen	1.11	185.2	20.4	416.2 ^a
Phenylbutazone	1.16	267.0	20.9	436.8 ^a
Phenacetin	1.22	146.5 ^a	22.2	492.8 ^b
Tolbutamide	1.23	229.0	22.3	497.3 ^a
Aspirin	1.40	128.4	24.5	600.3 ^a
Adipic acid	1.36	107.5 ^c	24.8 ^d	615.0
Sulfathiazole	1.55	164.7 ^c	25.1 ^d	639.0
Sulfadiazine	1.41	182	25.6 ^c	655.4
Paracetamol	1.28	118.6 ^c	26.1 ^d	681.2
Caffeine (anhydrous)	1.43	144	26.6 ^f	707.6
Theophylline (anhydrous)	1.44	124	28.6 ^f	818.0
Sucrose	1.56	219.4	32.8 ^b	1075.8
α-lactose monohydrate	1.53	235.5	35.0 ^g	1225.0
Anhydrous β-lactose	1.55	220.8	39.9 ^h	1592.0
Microcrystalline cellulose	1.51	–	25.7 ⁱ	660.5

^a Roberts et al. (1991).

^b Roberts et al. (1993a).

^c Molar volume calculated from true densities and molecular weight.

^d Calculated by same method as Roberts et al. (1991).

^e Martin et al. (1983).

^f Huu Phuoc et al. (1987).

^g Maeda et al. (1994).

^h Huu Phuoc et al. (1986).

ⁱ Roberts and Rowe (1993).

cules) are explored as a means of generating algorithms for expert system development.

2. Experimental

The materials examined in this study are listed in Table 1 together with their true densities (determined by air comparison pycnometry), molar volumes (calculated either from the group contribution method of Fedors, (1969) or from molecular weight and true density) and solubility parameters/cohesive energy densities. The latter were

either taken from the literature or calculated using group molar attraction constants (Barton, 1983).

Beams for the determination of tensile strength and critical stress intensity factor were prepared by compacting the powders in a rectangular die, 20 mm in length and 7 mm in breadth using a hydraulic press (Specac). The weight of powder was adjusted to give beam thicknesses of approx. 1 and 3 mm for tensile strength and K_{IC} (see Roberts et al. (1993b) for thickness criterion) measurements, respectively. The die was lubricated prior to compaction by applying a solution of stearic acid in methanol (2% w/w) using a paintbrush and left to dry. A series of beams were made to give a range of porosities (fractional void volumes), determined from the dimensions of the beams and densities of the materials.

2.1. Measurement of tensile strength, σ_{Ts}

The load, F , for fracture of the beams using three-point beam test was determined using a 30 kN tensometer (M30K, J.J. Lloyd Instruments) at a crosshead speed of 0.2 mm min⁻¹, the tensile strength, σ_{Ts} being subsequently calculated using Eq. 1:

$$\sigma_{Ts} = \frac{3Fl}{2wt^2} \quad (1)$$

where l is length, w denotes the width and t is the thickness of the beam.

The tensile strength at zero porosity, σ_T , was subsequently determined by extrapolating, σ_{Ts} , the specimen tensile strength at a known porosity, P , using the following equation as recommended by Ryshkewitch (1953) and Newton et al. (1993):

$$\sigma_{Ts} = \sigma_T \exp^{-bP} \quad (2)$$

where b is a constant.

2.2. Measurement of critical stress intensity factor, K_{IC}

The stress intensity factor of a specimen, K_{IC} , was calculated using Eq. 3 (Brown and Srawley,

1966) for three-point single-edge notched beam (SENB) testing:

$$K_{IC} = Y \frac{3Fln^{1/2}}{2wt^2} \quad (3)$$

where F is the load that caused the beam to fracture, n represents the notch length, l is the distance between rollers (17 mm), w denotes the width, t is the thickness, and Y is a function of the geometry of the specimen expressed as a polynomial of the parameter n/t (Brown and Srawley, 1966) given by Eq. 4:

$$Y = 1.93 - 3.07\left(\frac{n}{t}\right) + 14.53\left(\frac{n}{t}\right)^2 - 25.11\left(\frac{n}{t}\right)^3 + 25.8\left(\frac{n}{t}\right)^4 \quad (4)$$

Details of the full experimental method are given in a previous publication (Roberts et al., 1993).

The critical stress intensity factor at zero porosity, K_{IC} , was evaluated by extrapolating the specimen value, K_{ICs} , at porosity P using an exponential equation as suggested by Roberts and Rowe (1989), York et al. (1990) and Roberts et al. (1993b):

$$K_{ICs} = K_{IC} \exp^{-bP} \quad (5)$$

where b is a constant.

3. Results and discussion

3.1. Tensile strength

Data of tensile strength vs porosity for some representative materials are shown in Fig. 1. The data were extrapolated using the exponential relationship (Eq. 2). The regression constants and standard errors in the extrapolated values for each material are shown in Table 2. It might be expected that in, general, tensile strength will show more variability than other mechanical measurements because of the importance of the influence of cracks. This does not appear to be the case, since the variability is similar to that seen for other mechanical properties determined by beam bending (Roberts et al., 1991, 1993b).

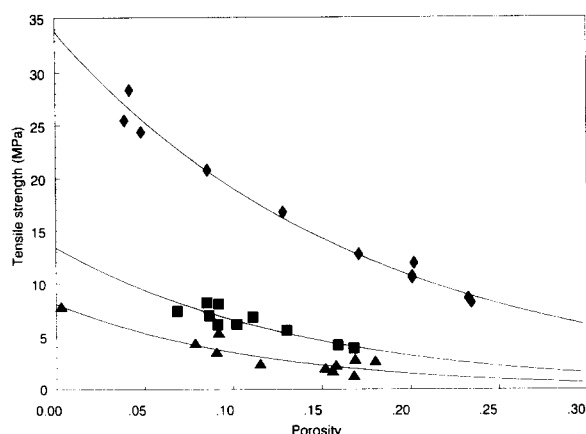


Fig. 1. Tensile strength vs porosity for various representative materials: (◆) microcrystalline cellulose, (■) theophylline, (▲) sulfadiazine.

The value of b has often been reported to be related to the material characteristics. Rice (1977) found that for a wide range of ceramics the value of b was 4 ± 2 (Young's modulus showed the same trend). Furthermore, low values of b were characteristic of homogeneous pore distribution, whereas higher values of b were indicative of inhomogeneous porosity. The lack of homogeneity in the specimens may be related to ease of deformation (e.g., materials with a higher yield stress would be less homogeneous than those

Table 2

Tensile strength, σ_T and b values from regression analysis (variation in tensile strength is indicated at \pm SE)

Material	σ_T (MPa)	b
Testosterone propionate	5.20 ± 1.00	6.515
Phenylbutazone	6.80 ± 1.16^a	—
Ibuprofen	7.71 ± 0.62	7.841
Sulfadiazine	8.04 ± 0.75	8.441
Tolbutamide	9.60 ± 1.36	8.510
Caffeine (anhydrous)	9.93 ± 1.06	3.529
Aspirin	11.89 ± 1.49	9.820
Theophylline (anhydrous)	13.33 ± 0.77	8.294
Paracetamol	13.38 ± 0.74	7.199
α -lactose monohydrate	18.33 ± 2.12	8.215
Avicel PH101	33.83 ± 1.03	5.749
Anhydrous β -lactose	35.69 ± 2.01	10.924

^a Represents the mean and standard deviation for eight beams ranging from 0 to 0.037 porosity.

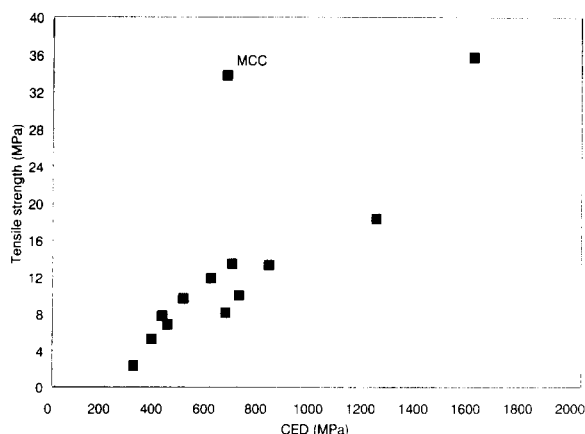


Fig. 2. Tensile strength vs cohesive energy density (continuous line represents Eq. 6).

with lower values) or to anisotropic behaviour of the deformation. The lowest values of b in Table 2 are represented by those materials with the lowest yield stress values, e.g., testosterone propionate, anhydrous caffeine and microcrystalline cellulose. Therefore, this suggestion might be generally applicable, although generally over the whole data set the trends are somewhat variable.

Fig. 2 shows the relationship between tensile strength at zero porosity and cohesive energy density (CED) for all twelve materials. Linear regression analysis on eleven of the materials (microcrystalline cellulose was excluded from the analysis) yielded an equation of the form:

$$\sigma_T = 0.0183 \pm 0.011 \text{CED} \quad (6)$$

with a correlation coefficient of 0.9463 (both variables having the units of MPa).

The constant of proportionality is an order of magnitude lower than that suggested by Gardon (1966) for the strength of metal whiskers and unoriented polymers (0.25). An analysis of Gardon's data for polymers shows a constant of proportionality of the order of 0.09 similar to that calculated for microcrystalline cellulose of 0.05. A theory describing the discrepancy between observed tensile strength and theoretically calculated tensile strengths was first proposed by Griffith (1920, 1924) who postulated that real solids contain fine cracks or microflaws and it is only at the tip of the crack that stress concentration effects cause the theoretical strength to be

reached. Away from the crack or flaw stresses are low and using, an energy balance Griffith was able to derive the equation:

$$\sigma_T = \left(\frac{E\gamma}{\pi c} \right)^{1/2} \quad (7)$$

where E and γ are Young's modulus and surface energy, respectively, of the material and c denotes the crack length.

It should be noted that since $(2/\pi)^{1/2}$ is approximately equal to unity, Eq. 7 is similar to that derived by Orowan (1949) for the prediction of maximum theoretical tensile strength.

$$\sigma_T = \left(\frac{E\gamma}{A_0} \right)^{1/2} \quad (8)$$

where A_0 is the equilibrium distance between two surfaces/atoms/molecules.

Hence, if cracks had initial lengths in the order of the equilibrium molecular distance, A_0 , maximum theoretical cohesive strengths could be approached. However, in reality, c is much larger than A_0 and therefore actual strengths of solids always tend to very much lower than the theoretical, as is found above for pharmaceutical solids.

3.2. Critical stress intensity factor

Table 3 shows data for critical stress intensity factors extrapolated to zero porosity for the 13

Table 3
Critical stress intensity factors, K_{IC} , and b values from regression analysis (variation in K_{IC} is indicated as \pm SE)

Material	K_{IC} (MPa m ^{1/2})	b
Ibuprofen ^a	0.104 ± 0.005	11.475
Paracetamol ^a	0.115 ± 0.014	6.893
Tolbutamide	0.113 ± 0.016	8.756
Sulfathiazole	0.127 ± 0.004	8.075
Phenylbutazone	0.140 ± 0.012	5.191
Adipic acid ^a	0.140 ± 0.009	11.148
Aspirin ^a	0.156 ± 0.009	9.579
Phenacetin	0.158 ± 0.011	9.884
Sulfadiazine	0.148 ± 0.014	7.794
Caffeine (anhydrous)	0.261 ± 0.009	6.527
Theophylline (anhydrous)	0.264 ± 0.009	8.267
Sucrose ^a	0.224 ± 0.011	12.135
α-lactose monohydrate ^a	0.354 ± 0.006	13.288

^a Measurements of K_{IC} from Roberts et al. (1993b).

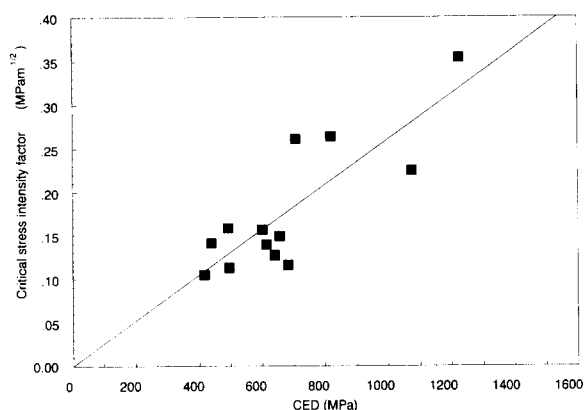


Fig. 3. Critical stress intensity factor vs cohesive energy density (continuous line represents Eq. 9).

materials. Data from a previous study (Roberts et al., 1993b) have been included to extend the number of materials. The standard errors are similar to those reported previously and the values of b are comparable to those in Table 2.

Fig. 3 shows the relationship between critical stress intensity factor at zero porosity and cohesive energy density for all 13 materials. Linear regression analysis yielded an equation of the form,

$$K_{IC} = 0.000260 \pm 0.000016CED \quad (9)$$

with a correlation coefficient of 0.8312.

Although still highly significant, the correlation is not as good as that for tensile strength. This not surprising since the units on either side of Eq. 9 are not consistent, ($\text{MPa m}^{1/2}$ for K_{IC} and MPa for CED).

For real materials Eq. 7 does not strictly apply, since plastic deformation usually accompanies the fracture process. Orowan (1949) and Irwin (1949) proposed a modification of the Griffith equation, i.e.

$$\sigma_T = \left(\frac{2E(\gamma + R)}{\pi c} \right)^{1/2} \quad (10)$$

where R is the plastic work or fracture toughness needed to be overcome if the crack is to increase and propagate.

The relationship between tensile strength and critical stress intensity factor follows from Eq. 10 since $R \gg \gamma$, i.e.

$$\sigma_T = \left(\frac{ER}{c} \right)^{1/2} \quad (11)$$

Since by definition:

$$K_{IC} = (ER)^{1/2} \quad (12)$$

then Eq. 11 becomes:

$$\sigma_T = \frac{K_{IC}}{c^{1/2}} \quad (13)$$

It has been assumed that for all the solids studied $c^{1/2}$ will be constant. However, this may not always be the case as there is no independent method of determining values of c , hence the inconsistency in Eq. 9. An approximate value for the flaw length, c , can be calculated from Eq. 13 substituting for σ_T using Eq. 6 and K_{IC} , using Eq. 9, giving a value in the region of 200 μm . This lies somewhat intermediate between flaws in glass agglomerates of 50% porosity, containing a polymer binder (between 280 and 1300 μm ; Mullier et al., 1987) and flaw sizes in titanium dioxide compacts (between 30 and 600 μm ; Kendall et al., 1987). This might be expected in view of the brittleness of pharmaceutical solids (Roberts et al., 1993b) which lie in the range intermediate between polymers and ceramics.

4. Conclusion

For all the materials studied the equations derived allow the prediction of tensile strengths and critical stress intensity factors to within 20% of the experimental values determined from beam bending. In view of the inherent unpredictable nature of flaw sizes (Eq. 13) this is considered to be satisfactory as the basis of algorithms for expert system development. These mechanical data, in addition to the Young's modulus of elasticity and indentation hardness/yield stress, will allow predictions to be made of the compaction and comminution behaviour of pharmaceutical materials purely from a knowledge of their chemical structure.

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